



COMPARISON ON THE AVERAGE MODEL OF A THREE PHASE INVERTER WITH ITS MODIFIED NODAL ANALYSIS METHOD

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Abstract

A comparison on the state space average model of a three phase three wire inverter with its modified nodal analysis method is presented in this paper. The equations formulated in the Modified Analysis Method end up with a linear system of equations which can be solved in one matrix inversion step. The rippled output waveforms from the nodal analysis can be eliminated by using the average model of a three phase inverter.

Index Terms:Average Model Method, Modified Nodal Analysis (MNA) Method, Three Phase Inverter and Transient Simulation Techniques.

I. INTRODUCTION

Voltage source inverters have been extensively studied in the last few decades in most industrial sectors for many applications. The application of three phase pulse width modulated power converters is rapidly growing beyond adjustable speed ac motor drives to include distributed generation systems, power quality conditioners, etc. By considering the increasing energy demands, power quality and efficiency; a control and power conversion using power electronics have become an important topic today. The effect of inverter output multiple-filter with nonlinear loads on the inverter input current and load voltage is discussed in [1]. A functional simulation model for the voltage-source inverter

(VSI) using the switching function concept is studied and implemented in [2]. An average inverter model operating in two complementary modes suitable for micro grid simulation applications is proposed in [3] by taking into account the nonlinear behavior of the switches, delays in the control loops, and the practical constraints. In [4], the author focuses on a combination of three phase (VSI) with a predictive current control to provide an optimized system for three-phase inverter that controls the load current.

With the advent of increased use of inverters, various switching models are established. The average method is shown to be an effective method for analysis and controller design in inverters [5] – [7]. Conventional switching models and state-space averaging methods which take dead-time effects into consideration have been applied successfully in the inverters [8], [9].

II. MODIFIED NODAL ANALYSIS OR MATRIX METHOD MODEL OF A THREE PHASE INVERTER

A basic three phase inverter is a six-step bridge inverter. It uses a minimum of six thyristors. In inverter terminology, a step is defined as a change in the firing from one thyristor to the next thyristor in the proper sequence. For one cycle of 360°, each step would be of 60° interval for a

six-step inverter. This means that the thyristors would be gated at regular intervals of 60° in proper sequence so that a three phase ac voltage is synthesized at the output terminals of a six-step inverter. The Fig. 1 shows the power circuit diagram of a three phase bridge inverter using six thyristors and six diodes.

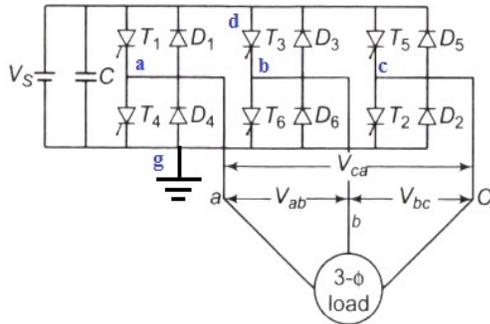


Fig. 1 Circuit Diagram of a three phase bridge inverter using thyristors

Presently, the use of IGBTs in single phase and three phase inverters is on the rise. The basic circuit configuration of inverter, however, remains unaltered with just a small change of replacing the thyristors with IGBTs. A large capacitor connected at the input terminals tends to make the input dc voltage constant. This capacitor also suppresses the harmonics fed back to the dc source. In the Fig. 1, commutation and snubber circuits are omitted for simplicity. The thyristors are numbered in the sequence in which they are triggered to obtain voltages v_{ab} , v_{bc} , v_{ca} at the output terminals a, b, c of the inverter. There are two possible patterns of gating the thyristors. In one pattern, each thyristor conducts for 180° and in the other, each thyristor conducts for 120° . But in both these patterns, gating signals are applied and removed at 60° intervals of the output voltage waveform [10].

Consider a three phase inverter with 180° conduction mode, where each thyristor conducts for 180° of a cycle.

The thyristor pair in each arm, i.e. T_1, T_4 ; T_3, T_6 and T_5, T_2 are turned on with a time interval of 180° . The thyristors in the upper and lower group conduct at an interval of 120° . The conduction time period of a 180° mode 3 phase inverter is as shown in the Fig. 2. It can be understood from the figure that in every step of 60° duration, only three thyristors are conducting – one from the upper group and two

from the lower group or two from the upper group and one from the lower group.

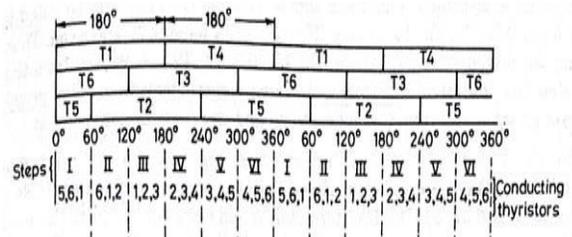


Fig. 2 Conduction period for a 180° mode 3-phase VSI

Each thyristor has an on-resistance value R_{on} , when the thyristor conducts and an off-resistance value R_{off} , when it is not conducting. For the analysis purpose, the three phase bridge inverter is replaced by its equivalent circuit diagram with its on-state and off-state resistance values. The equivalent circuit diagram is obtained for each conduction period, where three thyristors conduct, one thyristor from the upper group and two from the lower group or vice versa.

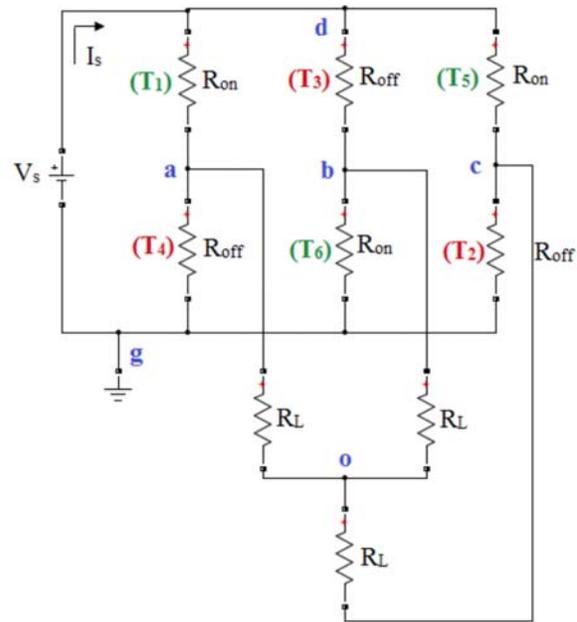


Fig. 3 Equivalent circuit diagram of a three phase bridge inverter for the interval $\omega t = 0$ to 60° (T_1, T_5 and T_6 conduct)

Consider the interval $\omega t = 0^\circ$ to 60° , where the thyristors T_1, T_5 and T_6 conduct. The conducting thyristors are replaced by their on-resistance values R_{on} and the rest of the thyristors with their off-resistance values R_{off} . The equivalent circuit diagram will be as in Fig. 3.

Now, applying KCL at the nodes a, b, c and d, we get the following equations.

At node 'a';

$$\frac{V_a - V_d}{R_{on}} + \frac{V_a}{R_{off}} + \frac{V_a - V_o}{R_L} = 0 \quad (1)$$

At node 'b';

$$\frac{V_b}{R_{on}} + \frac{V_b - V_d}{R_{off}} + \frac{V_b - V_o}{R_L} = 0 \quad (2)$$

At node 'c';

$$\frac{V_c}{R_{off}} + \frac{V_c - V_d}{R_{on}} + \frac{V_c - V_o}{R_L} = 0 \quad (3)$$

$$\frac{V_d - V_a}{R_{on}} + \frac{V_d - V_b}{R_{off}} + \frac{V_d - V_c}{R_{on}} + I_s = 0 \quad (4)$$

At node 'o'

$$\frac{V_o - V_a}{R_L} + \frac{V_o - V_b}{R_L} + \frac{V_o - V_c}{R_L} = 0 \quad (5)$$

Converting the above nodal equations into matrix form which amounts to solve a linear system $AX=Z$, where, A denotes the admittance matrix or the modified nodal analysis matrix, X denotes the unknown matrix (unknown node voltages and currents) and Z is the input matrix (source voltages). We can obtain the unknown matrix X as $X = A^{-1}Z$.

$$\begin{bmatrix} \frac{1}{R_{on}} + \frac{1}{R_{off}} + \frac{1}{R_L} & 0 & 0 & -\frac{1}{R_{on}} & -\frac{1}{R_L} & 0 \\ 0 & \frac{1}{R_{on}} + \frac{1}{R_{off}} + \frac{1}{R_L} & 0 & -\frac{1}{R_{off}} & -\frac{1}{R_L} & 0 \\ 0 & 0 & \frac{1}{R_{on}} + \frac{1}{R_{off}} + \frac{1}{R_L} & -\frac{1}{R_{on}} & -\frac{1}{R_L} & 0 \\ -\frac{1}{R_{on}} & -\frac{1}{R_{off}} & -\frac{1}{R_{on}} & \frac{2}{R_{on}} + \frac{1}{R_{off}} & 0 & 1 \\ -\frac{1}{R_L} & -\frac{1}{R_L} & -\frac{1}{R_L} & 0 & \frac{3}{R_L} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_o \\ I_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ V_d \\ 0 \\ 0 \end{bmatrix}$$

Assuming suitable values for the resistances as $R_{on} = 1m\Omega$, $R_{off} = 1M\Omega$, $R_L = 10k\Omega$ and substituting in the above matrix equation, the inverse of A matrix (A-1) can be obtained using an online inverse matrix calculator. The A-1 matrix (after substituting the resistance values) for the interval taken, $\omega t = 0$ to 60° can be evaluated as below.

$$A^{-1} = \begin{bmatrix} 0.000999 & 3.33*10^{-11} & 3.33*10^{-11} & 0 & 0.000333 & 0.999 \\ 3.33*10^{-11} & 0.000999 & 3.33*10^{-11} & 0 & 0.000333 & 6.766*10^{-8} \\ 3.33*10^{-11} & 3.33*10^{-11} & 0.000999 & 0 & 0.000333 & 0.999 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0.000333 & 0.000333 & 0.000333 & 0 & 3333.333 & 0.666 \\ 0.999 & 6.766*10^{-8} & 0.999 & 1 & 0.666 & -0.00006966 \end{bmatrix}$$

Similarly, A-1 matrix for each conducting interval can be obtained. The A-1 matrix and the Z matrix is given as input to the C program and the corresponding outputs waveforms are obtained.

During the interval, $\omega t = 60$ to 120° , the diodes T1, T2 and T6 will be conducting and the inverse matrix can be obtained as below.

$$A^{-1} = \begin{bmatrix} 0.000999 & 3.33*10^{-11} & 3.33*10^{-11} & 0 & 0.000333 & 0.999 \\ 3.33*10^{-11} & 0.000999 & 3.33*10^{-11} & 0 & 0.000333 & 3.433*10^{-8} \\ 3.33*10^{-11} & 3.33*10^{-11} & 0.000999 & 0 & 0.000333 & 3.433*10^{-8} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0.000333 & 0.000333 & 0.000333 & 0 & 3333.3336 & 0.333 \\ 0.999 & 3.433*10^{-8} & 3.433*10^{-8} & 1 & 0.333 & -0.00006966 \end{bmatrix}$$

During the interval, $\omega t = 300$ to 360° , the diodes T4, T5 and T6 will be conducting and the inverse matrix can be obtained as below.

$$A^{-1} = \begin{bmatrix} 0.000999 & 3.33*10^{-11} & 3.33*10^{-11} & 0 & 0.000333 & 3.433*10^{-8} \\ 3.33*10^{-11} & 0.000999 & 3.33*10^{-11} & 0 & 0.000333 & 3.433*10^{-8} \\ 3.33*10^{-11} & 3.33*10^{-11} & 0.000999 & 0 & 0.000333 & 0.999 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0.000333 & 0.000333 & 0.000333 & 0 & 3333.3336 & 0.333 \\ 3.433*10^{-8} & 3.433*10^{-8} & 0.999 & 1 & 0.333 & -0.00006966 \end{bmatrix}$$

III. AVERAGE MODEL OF A THREE PHASE INVERTER USING TRANSIENT SIMULATION TECHNIQUES

In the study of inverters and their dynamic performance, various switching models are obtained based on the switching conditions, as in the matrix method, which is feasible for a single inverter or a small scale inverter system.

Therefore, in case of large scale inverter systems, switching models becomes complicated. Hence, a state space averaging method is formulated which is effective for the analysis and controller design purpose in inverters. However, it is yet to give guaranteed high accuracy performance. A generalized state space averaging model is considered which enhances the fundamental voltage and current calculation with desired accuracy and precision. At the same time, this method enables the steady state as well as transient analysis processes [11]. The circuit diagram of a 3 phase 3 wire Voltage source inverter is as shown in the Fig. 4.

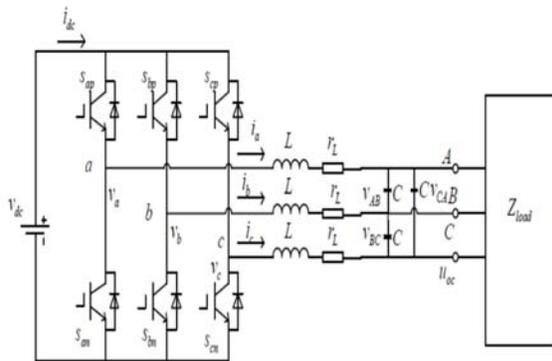


Fig. 4 Circuit diagram of a three phase three wire voltage source inverter

In the Fig. 4, assuming that the loads are 3 phase symmetrical resistive loads in delta connection, whose value are R, we obtain the equations from (6) to (11).

$$v_{ab} = v_a - v_b = L \frac{d(i_a - i_b)}{dt} + r_L(i_a - i_b) + V_{AB} \quad (6)$$

$$v_{bc} = v_b - v_c = L \frac{d(i_b - i_c)}{dt} + r_L(i_b - i_c) + V_{BC} \quad (7)$$

$$v_{ca} = v_c - v_a = L \frac{d(i_c - i_a)}{dt} + r_L(i_c - i_a) + V_{CA} \quad (8)$$

$$i_{ab} = C \frac{dV_{AB}}{dt} + \frac{1}{R} V_{AB} \quad (9)$$

$$i_{bc} = C \frac{dV_{BC}}{dt} + \frac{1}{R} V_{BC} \quad (10)$$

$$i_{ca} = C \frac{dV_{CA}}{dt} + \frac{1}{R} V_{CA} \quad (11)$$

Where, i_{ab}, i_{bc}, i_{ca}

are virtual line currents which can be calculated as mentioned in the equations from equation (12) to (17).

$$i_a = i_{ab} - i_{ca} \quad (12)$$

$$i_b = i_{bc} - i_{ab} \quad (13)$$

$$i_c = i_{ca} - i_{bc} \quad (14)$$

$$i_{ab} = \frac{1}{3}(i_a - i_b) \quad (15)$$

$$i_{bc} = \frac{1}{3}(i_b - i_c) \quad (16)$$

$$i_{ca} = \frac{1}{3}(i_c - i_a) \quad (17)$$

By the average model, it can be written that,

$$v_{ab} = v_a - v_b = (s_a - s_b).V_{dc} \quad (18)$$

$$v_{bc} = v_b - v_c = (s_b - s_c).V_{dc} \quad (19)$$

$$v_{ca} = v_c - v_a = (s_c - s_a).V_{dc} \quad (20)$$

Thus, the conventional state equations of three phase inverter can easily be constructed.

$$\frac{di_{ab}}{dt} = -\frac{r_L}{L} i_{ab} - \frac{1}{3L} V_{AB} + \frac{1}{3L} v_{ab} \quad (21)$$

$$\frac{di_{bc}}{dt} = -\frac{r_L}{L} i_{bc} - \frac{1}{3L} V_{BC} + \frac{1}{3L} v_{bc} \quad (22)$$

$$\frac{di_{ca}}{dt} = -\frac{r_L}{L} i_{ca} - \frac{1}{3L} V_{CA} + \frac{1}{3L} v_{ca} \quad (23)$$

$$\frac{dV_{AB}}{dt} = -\frac{1}{RC} V_{AB} + \frac{1}{C} i_{ab} \quad (24)$$

$$\frac{dV_{BC}}{dt} = -\frac{1}{RC} V_{BC} + \frac{1}{C} i_{bc} \quad (25)$$

$$\frac{dV_{CA}}{dt} = -\frac{1}{RC} V_{CA} + \frac{1}{C} i_{ca} \quad (26)$$

The differential voltage and current equations can be converted to its equivalent Forward Euler form and substituting the equations (18) to (20) in the Forward Euler equations we obtain the value for the (n+1)th value of the corresponding ac voltage and ac current.

$$i_{ab}^{n+1} = h * \left[-\frac{r_L}{L} i_{ab} - \frac{1}{3L} V_{AB} + \frac{1}{3L} (s_a - s_b).V_{dc} \right] + i_{ab} \quad (27)$$

$$i_{bc}^{n+1} = h * \left[-\frac{r_L}{L} i_{bc} - \frac{1}{3L} V_{BC} + \frac{1}{3L} (s_b - s_c).V_{dc} \right] + i_{bc} \quad (28)$$

$$i_{ca}^{n+1} = h * \left[-\frac{r_L}{L} i_{ca} - \frac{1}{3L} V_{CA} + \frac{1}{3L} (s_c - s_a).V_{dc} \right] + i_{ca} \quad (29)$$

$$V_{AB}^{n+1} = \frac{h}{C} i_{ab} + V_{AB} \quad (30)$$

$$V_{BC}^{n+1} = \frac{h}{C} i_{bc} + V_{BC} \quad (31)$$

$$V_{CA}^{n+1} = \frac{h}{C} i_{ca} + V_{CA} \quad (32)$$

The switching function is given by:

$$s_k = \frac{m}{2} \cos(\omega t - \phi_k) + \frac{1}{2} \quad (33)$$

Where $k = a, b, c$; m is the modulation ratio taken as 0.6

$$\phi_a = \phi_0, \phi_b = \phi_0 + \frac{2\pi}{3}, \phi_c = \phi_0 - \frac{2\pi}{3}$$

Where ϕ_0 is the initial phase angle.

IV. RESULTS AND DISCUSSIONS

The output waveform obtained in the MNA method of modelling of inverter with parameter specifications as $R_{on} = 1m\Omega$, $R_{off} = 1M\Omega$ and $RL = 10k\Omega$ is shown in the Fig. 5. The figure depicts the phase voltage in the MNA method with input amplitude of 70 V.

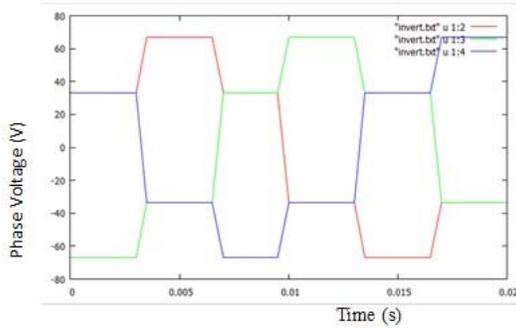


Fig. 5 Inverter output phase voltage waveform in matrix method (X axis: 1 unit = 0.005s & Yaxis: 1 unit = 20V)

In the case of the average model of a three phase inverter, the parameter specifications used for simulation are as in the Table I.

Table I: Parameters for simulation

| SYMBOL | PARAMETERS | VALUE | UNIT |
|----------|---------------------------|-------|----------|
| V_{dc} | Input Voltage | 622.5 | V |
| m | Modulation Ratio | 0.6 | - |
| f | Fundamental frequency | 50 | Hz |
| ϕ_0 | Initial phase angle | 0 | rad/s |
| L | Filtering Inductor | 1.0 | mH |
| r_L | ESR of filtering inductor | 1.245 | Ω |
| C | Filter Capacitor | 1.0 | mF |
| R | Load Resistance | 10 | Ω |
| h | Time step | 100 | μs |

The simulation waveform for phase currents at the output of the inverter is as shown in the Fig. 6. The figure shows the ac phase currents with a magnitude of 130 A for the given input dc of 622.5 V

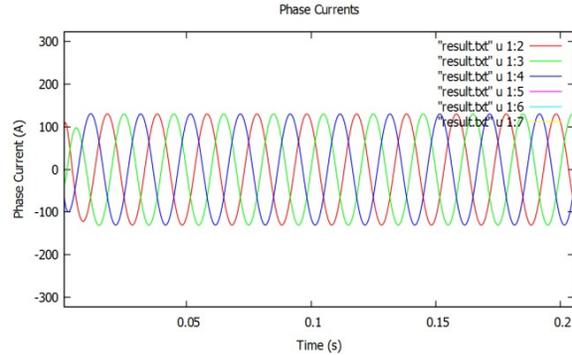


Fig. 6 Output phase current waveform for the dc input of 622.5 V (X axis: 1 unit = 0.05s and Y axis: 1 unit = 100 A)

The simulation waveform for phase voltages at the output of the inverter is as shown in the Fig. 7. The figure shows the ac phase voltages with a magnitude of $(2V_{dc}/3 = 415 V)$ for the given input dc of 622.5 V.

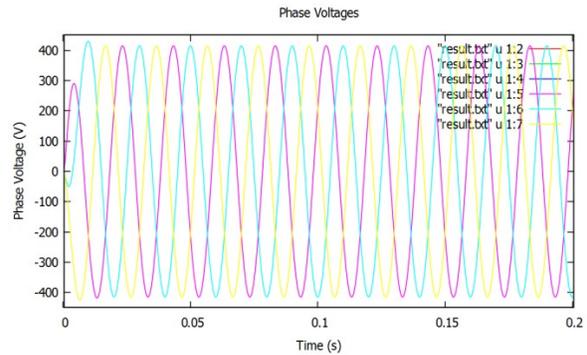


Fig. 7 Output phase voltage waveform for the dc input of 622.5 V (X axis: 1 unit = 0.05s and Y axis: 1 unit = 100 V)

V. CONCLUSION

In this paper, the modeling of three-phase three-wire inverter is done based on the Modified Nodal Analysis method and in the state space average method. The simulation waveforms obtained from the two methods are compared. It is observed from the paper that the average model of the inverter is more effective and efficient as it gives ripple free waveforms, thus the use of filters for smoothening the ripples can be eliminated.

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