HYDROMAGNETIC FREE CONVECTION FLOW OVER A TRUNCATED CONE EMBEDDED IN A THERMALLY STRATIFIED MEDIUM

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Abstract

The purpose of this paper is to present the effect of magnetic field and stratification on steady free convection flow of an incompressible electrically conducting fluid over a truncated cone embedded in a thermally stratified medium. The system of nonlinear partial differential equations is solved using an implicit finite difference scheme in combination with quasilinearisation technique. The effects of the governing parameters like magnetic and stratification parameter on the velocity, temperature, heat transfer and skin friction coefficients have been discussed. It is found that both magnetic stratification parameter retards the motion of the fluid flow which leads to the decrease in the velocity of the fluid. Also, the governing parameters have significant effect on thermal boundary layer.

Index Terms: MHD, Free convection, Stratified medium, Skin friction, Heat transfer.

I. INTRODUCTION

Natural convection is caused by the buoyancy force, obtained by the temperature difference of the fluid, when get contacted with heated surface at different locations. The study of natural convection flows is of much importance because of numerous science and engineering applications such as cooling of nuclear reactors, design of spacecrafts, design of solar energy collectors, power transformers, and steam generators, atmospheric and oceanic circulations. Stratification of the fluid arises due to temperature variation or due to the presence of different fluids. Free convection flows in a thermally stratified medium play an important role in practical applications like elimination of heat into the atmosphere for instance lakes, rivers and the seas, thermal power storage method such as solar ponds and heat transfer from thermal sources such as the condensers of power plants. The problem of free convection flow over a cone embedded in a stratified medium has been studied by Tripathi.et.al [1]. Chamkha [2] studied the effects of magnetic field and heat generation/absorption on natural convection from an isothermal surface in a stratified environment. Bapuji Pullepu.et.al [3] analysed transient free convective flow over a vertical cone embedded in a thermally stratified medium.

The flow of an electrically conducting fluid in the presence of a transverse magnetic field has varieties of applications for example MHD generators, pumps, accelerators, flow meters, grain storage and nuclear waste disposal. Since the convection currents are suppressed by Lorentz force, which is generated by the external magnetic field, it is used as control mechanism. Also, when the fluid is electrically conducting, the free convection flow is significantly controlled by an imposed magnetic field. Effect of thermo-physical quantities on the natural convection flow of gases over a vertical cone have been considered by Takhar.et.al [4]. Bapuji Pullepu and Chamkha [5] focused on transient laminar MHD free convective flow past a vertical cone with non-uniform surface heat flux. Other investigators [6-8] have also carried out the study of natural convection flow over the cone under the influence of magnetic field. Recently, Elbashbeshy.et.al [9] studied the effect of thermal radiation on free convection flow and heat transfer over a truncated cone in presence of pressure work and heat generation / absorption, without considering magnetic field effects.

The study of free convection flow over a truncated cone embedded in a thermally stratified medium in the presence of transverse magnetic field is not attempted so far. This motivates the author to analyze the effect of transverse magnetic filed on the steady laminar free convection flow over a cone embedded in a stratified medium.

II. GOVERNING EQUATIONS

The physical model and coordinate system of the problem is shown in Fig.1. The vertex of the cone is placed at the origin of the coordinate system, $x-$axis measures the distance along the surface of the body from the apex and $y-$axis measures distance normally outward. Consider the steady free convection boundary layer flow over a truncated cone situated in a thermally stratified medium.

The boundary layer is assumed to develop at the leading edge of the truncated cone $(x-x_0)$ which implies that the temperature at the circular base is assumed to be the same as the ambient temperature T_∞. The surface of the cone is maintained at a constant temperature T_w and the temperature of the fluid far from the body surface is given by $T_\infty(\bar{x}) = T_{\infty 0} + a\,\bar{x}$, where $a = (dT_\infty / d\bar{x}) > 0$ and $\bar{x}(= x^* \cos\gamma)$ is measured from apex of the cone and is parallel to the direction of the gravity and $T_{\infty 0}$ is the temperature of the fluid at the vertex of the cone. We also assume that $T_w > T_{\infty 0}$. The effect of stratification can be expressed in terms of a stratification parameter S, defines as $S = [a\,x^* \cos\gamma] / \Delta T_w$, [1] where x^* is the slant height of the cone and ΔT_w is the temperature difference, $T_w - T_\infty$. A magnetic field of strength B_0 is applied normal to the surface of the cone and it is assumed that magnetic Reynolds number of the flow is small enough so that the induced magnetic field is negligible. The fluid is assumed to have constant physical properties except for the density variation which is assumed to be important in buoyancy terms.

Under these assumptions, the boundary layer equations with Boussinesq approximation, governing the flow over a cone are:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta\cos\gamma(T - T_\infty) - \frac{\sigma B_0^{\,2} u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

subject to the boundary conditions

$$\left.\begin{array}{ll} x \ge 0: & u=0,\ v=0,\ T=T_w \ \text{ at } \ y=0 \\ & u=0,\ T=T_\infty(x) \ \text{ as } \ y\to\infty \\ x<0: & u=0,\ T=T_\infty(x) \ \text{ for all } \ y \end{array}\right\} \tag{4}$$

The assumptions considered in developing the Eqn's (1)-(3) holds in excellent if the boundary layer is assumed thin compared to the local radius of the cone. The local radius to a point in the boundary layer can be replaced by the radius of the truncated cone r, $r = x\sin\gamma$, where γ is semi vertical angle of the cone.

Eqns. (1) – (4) is valid in the domain $x_0 \le x \le \infty$, where x_0 is the leading edge of the truncated cone measured from the origin. u and v are the components of fluid velocity in the x and y directions respectively; ν is the kinematic viscosity; g is the gravitational acceleration; β is the coefficient of thermal expansion; α is the thermal diffusivity of the fluid; σ is the electric conductivity; ρ is the density of the fluid. The subscripts ∞ and w denote the conditions at the free stream and at the wall, respectively.

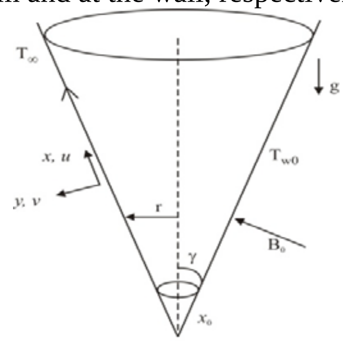


Fig.1. Physical model and coordinate system

On introducing the following similarity variables:

$$\psi = \nu\, r\left(Gr_{x*}^{1/4}\right) f(\eta,\xi); \quad \eta = \frac{y}{x*}\left(Gr_{x*}^{1/4}\right); \quad r\,u = \frac{\partial \psi}{\partial y};$$

$$r\,v = -\frac{\partial \psi}{\partial x}; \ \xi = \frac{x*}{x_0} = \frac{x - x_0}{x_0}; \quad G = \frac{T - T_\infty}{T_w - T_{\infty 0}}; \tag{5}$$

$$Gr_{x*} = \frac{g\beta \cos\gamma (T_w - T_{\infty 0}) x*^3}{\nu^2}$$

into the Eqn's (1)-(3), it is observed that continuity equation is automatically satisfied and Eqn's (2) and (3) reduce, respectively to

$$F'' + \left(\frac{3}{4} + \frac{\xi}{1+\xi}\right) f\,F' - \frac{1}{2}F^2 + G - MF = \xi\left(FF_\xi - F'\,f_\xi\right) \tag{6}$$

$$\mathrm{Pr}^{-1} G'' + \left(\frac{3}{4} + \frac{\xi}{1+\xi}\right) f\,G' - S(1+S)^{-1}(F+G) = \xi\left(FG_\xi - G'\,f_\xi\right) \tag{7}$$

where

$$u = \frac{\nu (Gr_{x*})^{1/2}}{x*} f'; \quad M = \frac{\sigma B_0^2 (x*)^2}{\rho \nu (Gr_{x*})^{1/2}};$$

$$F = f' = \frac{\partial f}{\partial \eta}; \quad \mathrm{Pr} = \frac{\nu}{\alpha} \tag{8}$$

$$v = -\frac{\nu (Gr_{x*})^{1/4}}{x*}\left[\left(\frac{\xi}{\xi+1} + \frac{3}{4}\right) f' + \xi\, f_\xi - \frac{\eta\, f'}{4}\right]$$

The transformed boundary conditions are:

$$\begin{array}{lll} F = 0, & G = 1 - S(1+S)^{-1}\,\xi(1+\xi)^{-1} & \text{at} \quad \eta = 0 \\ F = 0, & G = 0 & \text{as} \quad \eta \to \infty \end{array} \tag{9}$$

Here ψ and f are dimensional and dimensionless stream function respectively; η is the pseudo similarity variable; $(\,'\,)$ denotes the derivative with respect to η; M is the magnetic parameter; F and G are dimensionless velocity and temperature of the fluid; Pr is the Prandtl number; Gr_{x*} is the local Grashof number.

The skin friction and heat transfer in the form of Nusselt number can be expressed as

$$C_f = \frac{2(x*)^2\, \tau_w}{(Gr_{x*})\,\rho} \quad \text{and} \quad Nu = \frac{-x*\, q_w}{\Delta T_w} \tag{10}$$

where $\tau_w = \mu\left(\frac{\partial \mathrm{u}}{\partial \mathrm{y}}\right)$ and $q_w = \left(\frac{\partial \mathrm{T}}{\partial \mathrm{y}}\right)$

are the shear stress and rate of heat flux at the surface, respectively, where μ is dynamic viscosity. using (5) and (8) in (10), the skin friction and nusselt number can be written as

$$C_f\,(Gr_{x*})^{1/4} = 2\,(F')_{\eta=0} \quad \text{and}$$

$$Nu\,(Gr_{x*})^{-1/4} = -(1+S)(G')_{\eta=0} \tag{11}$$

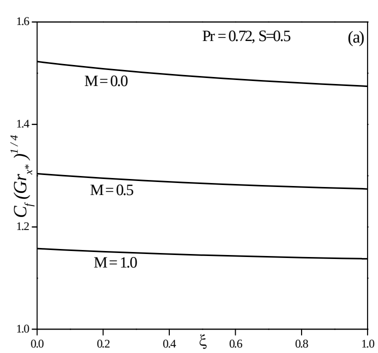
III. RESULTS AND DISCUSSION

Using implicit finite difference scheme along with quasilinearisation technique [10], we have solved the coupled non-linear partial differential equations (6) and (7) with two independent variables subject to the boundary conditions (9). For the sake of brevity the method is not explained here. The non-similar solutions have been obtained for various values of governing parameters viz., magnetic parameter (M), stratification parameter (S) for $\mathrm{Pr} = 0.72$ (air). With a view to asses the accuracy of our results, the results of skin friction and heat transfer parameter are compared with those of Elbashbeshy [9] in the absence of magnetic field and stratification parameter. The comparison results are presented in Table I and are found to be in good agreement with those of Elbashbeshy [9].

Effect of magnetic parameter (M) on skin friction $[C_f(Gr_{x*})^{1/4}]$ and heat transfer $[Nu(Gr_{x*})^{-1/4}]$ coefficients when $S = 0.5$ (stratification parameter) are illustrated in Fig.2. It is seen that an increase in magnetic parameter ($0 \le M \le 1$) causes a decrease in both $C_f(Gr_{x*})^{1/4}$ and $Nu(Gr_{x*})^{-1/4}$. The percentage of decrease in $C_f(Gr_{x*})^{1/4}$ at the stream wise location $\xi = 0.5$ for M varying from 0.0 to 1.0 is 34.7% and the percentage of decrease in $Nu(Gr_{x*})^{-1/4}$ is 17.8% for the same value of ξ in the range of $0 \le M \le 1$.

TABLE I
Comparision of skin friction and heat transfer parameter results for $M = 0.0$, $S = 0.0$ with those of Elbashbeshy [9]

Pr	Present results $-G'(0)$	Elbashbeshy [9] $-G'(0)$	Present results $F'(0)$	Elbashbeshy [9] $F'(0)$
0.1	0.1620	0.1627	1.1181	1.2151
0.7	0.3574	-	0.9403	-
1.0	0.4014	0.4010	0.8970	0.9082
10.0	0.8218	0.8268	0.5892	0.5928



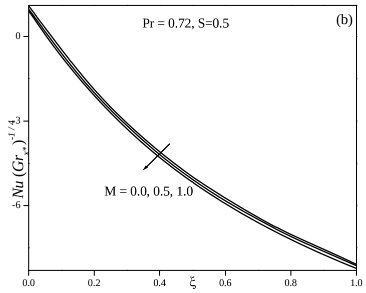


Fig. 2. Effect of magnetic parameter on (a) skin friction and (b) heat transfer coefficients

In Fig.3, corresponding velocity (F) and temperature (G) profiles are plotted for different values of M with $S = 0.5$ and $\xi = 0.5$. We observe that there is an increase in temperature profiles and a decrease in velocity profiles as M increases, this is due to the fact that the magnetic field applied normal to the flow of an electrically conducting fluid creates a drag force called Lorentz force which has a tendency to slow down the motion of the fluid. Both momentum and thermal boundary layer thickness increases with an increase in magnetic parameter M. Also, it is observed that as η increases velocity profiles (F) increases and reaches its maximum

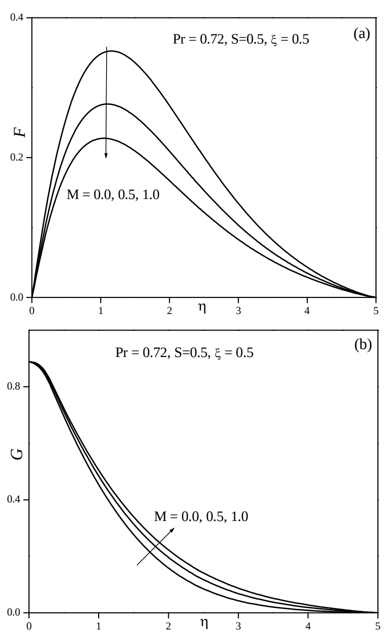


Fig. 3. Velocity and temperature profiles for different values of M

value at $\eta = 1.3$ and gradually decreases to the free stream conditions whereas, temperature profiles (G) decrease along the $\eta -$ direction owing to the velocity and temperature profiles of natural convection boundary layer flows.

Fig.4 depicts the effect of stratification parameter (S) on skin friction and heat transfer coefficients $[C_f (Gr_{x*})^{1/4},\ Nu(Gr_{x*})^{-1/4}]$ for a fixed value of magnetic parameter $M = 0.5$. It is clear from the figure that stratification parameter reduces buoyancy force, which results in decrease of both skin friction and heat transfer coefficients. The percentage of decrease in $C_f (Gr_{x*})^{1/4}$ is about 33% in the range of $0.0 \le S \le 1.0$ near $\xi = 0.5$. The effect of stratification on $Nu(Gr_{x*})^{-1/4}$ is more pronounced at higher stream wise locations, for instance at $\xi = 0.2$ the percentage of decrease in $Nu(Gr_{x*})^{-1/4}$ is about 429% when S increases from 0.0 to 1.0 while at $\xi = 0.4$ it is about 864% for the same change of stratification parameter [See Fig.4(b)]. The velocity (F) and temperature (G) profiles for various stratification parameter ($0.0 \le S \le 1.0$) under the influence of magnetic field are exhibited in Fig.5. We notice that the momentum boundary layer thickness increase, whereas thermal boundary layer thickness decrease for the increasing values of S. This shows that there is reduction in both velocity and temperature profiles. Further, a slight overshoot is observed in the temperature profiles which relates to the mild increase in heat transfer rate at the leading edge.

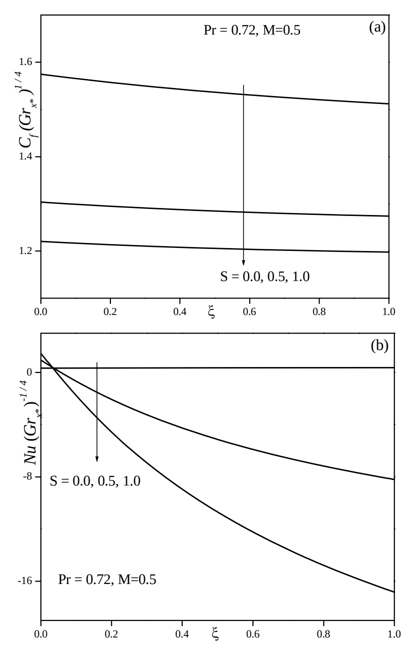


Fig. 4. Variation of (a) skin friction and (b) heat transfer coefficients with ξ for different values of S

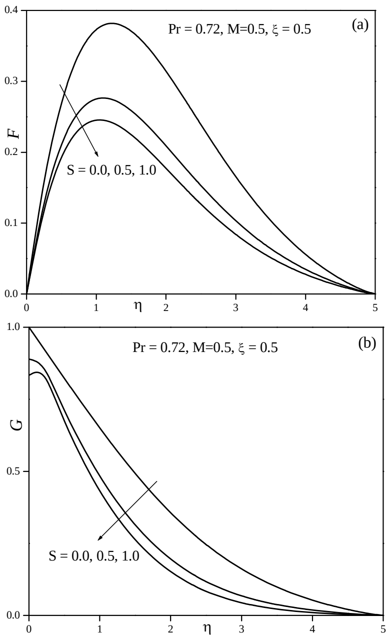


Fig. 5. Effect of stratification parameter on (a) velocity and (b) temperature profiles

IV. CONCLUSIONS

The laminar steady MHD free convection flow over a truncated cone embedded in a thermally stratified medium has been studied numerically. The effect of magnetic and stratification parameter have been considered on the flow and heat transfer characteristics. The skin friction coefficient decrease with the increase of both magnetic and stratification parameters, which in turn reduce the velocity of the fluid. As stratification parameter increases there is a decrease in temperature profiles whereas opposite trend is observed for an increase in magnetic parameter.

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